Homework Set 8

(sect 4.4 – 4.7)

1. Find the vector $\boldsymbol{x}$ determined by the coordinate vector $[\boldsymbol{x}]_{\mathcal{B}} = \begin{bmatrix} -4 \\ 8 \\ -7 \end{bmatrix}$ and the basis $\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}$

2. Find the coordinate vector $[\boldsymbol{x}]_{\mathcal{B}}$ of $\boldsymbol{x} = \begin{bmatrix} 3 \\ -5 \\ 4 \end{bmatrix}$ relative to the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \right\}$

3. Find the change-of-coordinates matrix from $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 7 \end{bmatrix} \right\}$ to the standard basis in $\mathbb{R}^3$.

4. The set $\mathcal{B} = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\boldsymbol{p}(t) = 3 + t - 6t^2$ relative to $\mathcal{B}$.

5. Use coordinate vectors to test the linear independence of the set of polynomials: $(1-t)^3$, $(2-3t)^2$, $3t^2-4t^3$. Explain your work and your conclusions.

For questions 6 through 8, (a) find a basis, and (b) state the dimension.

6. $\left\{\begin{bmatrix} 4s \\ -3s \\ -t \end{bmatrix} : s \text{ and } t \text{ in } \mathbb{R}\right\}$

7. $\left\{\begin{bmatrix} 3a+6b-c \\ 6a-2b-2c \\ -9a+5b+3c \\ -3a+b+c \end{bmatrix} : a, b, \& \ c \text{ in } \mathbb{R}\right\}$

8. $\{(a,b,c,d) : a-3b+c=0\}$

9. Determine the dimensions of $Nul(A)$ and $Col(A)$, where $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & 7 \\ 0 & 0 & 5 \end{bmatrix}$

10. The first four Laguerre polynomials are $1, 1-t, 2-4t+t^2, and\ 6-18t+9t^2-t^3$. Show that these polynomials form a basis of $\mathbb{P}_3$.

For questions 11 and 12, assume the matrix A is row equivalent to B. Without calculations, list rank(A) and dim(Nul(A)). Then find bases for Col(A) , Row(A), and Nul(A).

11. $A = \begin{bmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

12. $A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

13. If a 6×3 matrix A has rank 3, find $dim(Nul\ A)$, $dim(Row\ A)$, and $rank(A^T)$.

14. Let $\mathcal{B} = \{\boldsymbol{b_1}, \boldsymbol{b_2}\}$ and $\mathcal{C} = \{\boldsymbol{c_1}, \boldsymbol{c_2}\}$ be bases for a vector space V, and suppose $\boldsymbol{b_1} = -\boldsymbol{c_1} + 4\boldsymbol{c_2}$ and $\boldsymbol{b_2} = 5\boldsymbol{c_1} - 3\boldsymbol{c_2}$.

    a. Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

    b. Find $[\boldsymbol{x}]_{\mathcal{C}}$ for $\boldsymbol{x} = 5\boldsymbol{b_1} + 3\boldsymbol{b_2}$

15. Let $\mathcal{B} = \{\boldsymbol{b_1}, \boldsymbol{b_2}\}$ and $\mathcal{C} = \{\boldsymbol{c_1}, \boldsymbol{c_2}\}$ be bases for $\mathbb{R}^2$. Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ and the change-of-coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$. where $b_1 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, b_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$ and $c_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, c_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

16. In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis $\mathcal{B} = \{1 - 3t^2, 2 + t - 5t^2, 1 + 2t\}$ to the standard basis $\mathcal{C} = \{1, t, t^2\}$. Then write t^2 as a linear combination fo the polynomials in $\mathcal{B}$.